Strictly Confidential — (For Internal and Restricted Use Only)

Secondary School Certificate Examination

March 2017

Marking Scheme — Mathematics 30/1/1, 30/1/2, 30/1/3 [Delhi Region]

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

5. A full scale of marks - 0 to 90 has to be used. Please do not hesitate to award full marks if the answer deserves it.

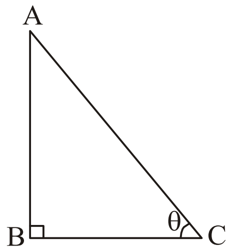
6. Separate Marking Scheme for all the three sets has been given.

7. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 30/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. 

$\tan\theta = \frac{AB}{BC} = \frac{\sqrt{3}}{1}$ $\frac{1}{2}$

$\Rightarrow \quad \theta = 60°$ $\frac{1}{2}$

2. $\frac{2}{3}\pi r^3 = 3\pi r^2 \Rightarrow r = \frac{9}{2}$ units $\frac{1}{2}$

$\therefore \quad d = 9$ units $\frac{1}{2}$

3. Favourable outcomes are –1, 0, 1 $\frac{1}{2}$

$\therefore \quad$ Required Probability $= \frac{3}{7}$ $\frac{1}{2}$

4. $\sqrt{(4-1)^2 + (k-0)^2} = 5$ $\frac{1}{2}$

$\Rightarrow \quad k = \pm 4$ $\frac{1}{2}$

SECTION B

5. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$ 1

$\Rightarrow \quad (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\Rightarrow \quad x = \frac{-5}{\sqrt{2}}, -\sqrt{2}$

or $\quad \frac{-5\sqrt{2}}{2}, -\sqrt{2}$ 1

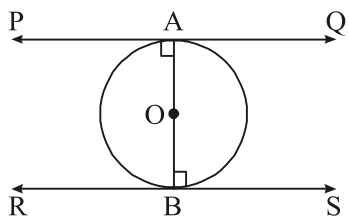
6. A.P. formed is 208, 216, 224, ..., 496 1

$a_n = 496$

$\Rightarrow$ $208 + (n - 1) \times 8 = 496$ $\frac{1}{2}$

$\Rightarrow$ $n = 37$ $\frac{1}{2}$

7.



$\angle PAO = \angle OBS = 90°$ 1

But these are alternate interior angles

$\therefore$ $PQ \parallel RS$ 1

8. $x^2 + k(2x + k - 1) + 2 = 0$

$\Rightarrow$ $x^2 + 2kx + (k^2 - k + 2) = 0$ $\frac{1}{2}$

For equal roots, $b^2 - 4ac = 0$

$\Rightarrow$ $4k^2 - 4k^2 + 4k - 8 = 0$ 1

$\Rightarrow$ $k = 2$ $\frac{1}{2}$

9. Correct construction 2

10. PA = PC + CA = PC + CQ

$\Rightarrow$ 12 = PC + 3 $\Rightarrow$ PC = 9 cm 1

PD = 9 cm

$\therefore$ PC + PD = 18 cm 1

SECTION C

11. $a_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$ ...(1) $\frac{1}{2}$

$a_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m}$ ...(2) $\frac{1}{2}$

Solving (1) and (2), $a = \frac{1}{mn}$ and $d = \frac{1}{mn}$ 1

$$S_{mn} = \frac{mn}{2}\left[2 \times \frac{1}{mn} + (mn - 1) \times \frac{1}{mn}\right]$$

$$= \frac{1}{2}(mn + 1)$$ 1

12. $S_n = \left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ...$ upto n terms

$$= \underbrace{(4 + 4 + ... + 4)}_{\text{n times}} - \frac{1}{n}(1 + 2 + 3 + ... + n)$$ 1

$$= 4n - \frac{1}{n} \times \frac{n(n + 1)}{2}$$ $\frac{1}{2} + 1$

$$= \frac{7n - 1}{2}$$ $\frac{1}{2}$

13. $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

For equal roots, $B^2 - 4AC = 0$ $\frac{1}{2}$

$\Rightarrow \quad 4m^2c^2 - 4(1 + m^2)(c^2 - a^2) = 0$ 1

$\Rightarrow \quad m^2c^2 - c^2 - m^2c^2 + a^2 + m^2a^2 = 0$ 1

$\Rightarrow \quad c^2 = a^2(1 + m^2)$ $\frac{1}{2}$

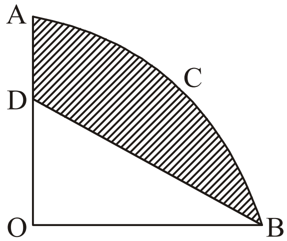
14. $\frac{3}{4} \times$ Volume of conical vessel = Volume of cylindrical vessel 1

Let the height of cylindrical vessel be h

$\Rightarrow \quad \frac{3}{\cancel{4}} \times \frac{1}{3} \times \pi \times 5 \times 5 \times \cancel{24}^{6} = \pi \times 10 \times 10 \times h$ 1

$\Rightarrow \quad h = \frac{3}{2}$ cm or 1.5 cm 1

15.

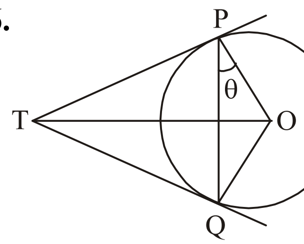


Area of shaded region = Area of quadrant OACB – Area of ΔODB 1

$= \left(\frac{22}{7} \times \frac{3.5 \times 3.5}{4} - \frac{1}{2} \times 3.5 \times 2\right) \text{cm}^2$ 1

$= \frac{49}{8}$ or 6.125 cm^2 1

16.



Let $\angle OPQ = \theta$

$\Rightarrow \quad \angle TPQ = 90° - \theta = \angle TQP$ 1

$\angle TPQ + \angle TQP + \angle PTQ = 180°$

$\Rightarrow \quad 90° - \theta + 90° - \theta + \angle PTQ = 180°$ $1\frac{1}{2}$

$\Rightarrow \quad \angle PTQ = 2\theta$

$= 2\angle OPQ$ $\frac{1}{2}$

17. A(–2, 0), B(2, 0), C(0, 2)

AB = 4 units, BC = $2\sqrt{2}$ units, AC = $2\sqrt{2}$ units 1

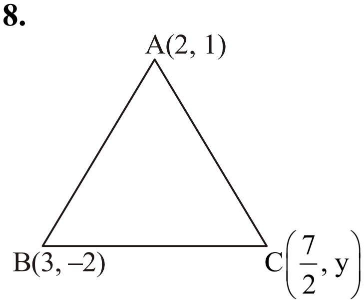
P(–4, 0), Q(4, 0), R(0, 4)

PQ = 8 units, QR = $4\sqrt{2}$ units, PR = $4\sqrt{2}$ units 1

$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{1}{2}$ 1

$\therefore \quad \Delta ABC \sim \Delta PQR$

18.



ar(ΔABC) = 5 sq.units

$\Rightarrow \quad \frac{1}{2}\left[2(-2-y) + 3(y-1) + \frac{7}{2}(1+2)\right] = 5$ $1\frac{1}{2}$

$\Rightarrow \quad y + \frac{7}{2} = 10$ 1

$\Rightarrow \quad y = \frac{13}{2}$ $\frac{1}{2}$

19. Total number of outcomes = 36

(i) Favourable outcomes are

(1, 1,) (1, 2) (1, 3) (1, 4) (1, 5) (2, 1) (2, 2) (2, 3)

(2, 4) (3, 1) (3, 2) (3, 3) (4, 1) (4, 2) (5, 1) i.e., 15

$\therefore$ P(sum less than 7) = $\frac{15}{36}$ or $\frac{5}{12}$ 1

(ii) Favourable outcomes are

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3)

(2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (4, 1)

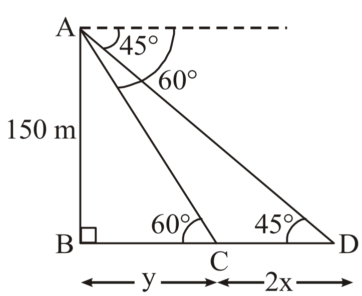
(4, 2) (4, 3) (5, 1) (5, 2) (5, 3) (6, 1) (6, 2) i.e., 25

P(product less than 16) = $\frac{25}{36}$ 1

(iii) Favourable outcomes are

$\therefore$ P(doublet of odd number) = $\frac{3}{36}$ or $\frac{1}{12}$ 1

20.



Correct Figure $\frac{1}{2}$

Let the speed of boat be x m/min

$\therefore$ CD = 2x

$$\frac{150}{y} = \tan 60° \Rightarrow y = \frac{150}{\sqrt{3}} = 50\sqrt{3}$$ 1

$$\frac{150}{y + 2x} = \tan 45° \Rightarrow 150 = 50\sqrt{3} + 2x$$

$\Rightarrow$ $x = 25(3 - \sqrt{3})$ 1

$\therefore$ Speed = $25(3 - \sqrt{3})$ m/min

= $1500(3 - \sqrt{3})$ m/hr. $\frac{1}{2}$

SECTION D

21. Correct construction of given triangle 2

Correct construction of similar triangle 2

22. Correct figure, given, to prove and construction $\frac{1}{2}\times 4=2$

Correct proof 2

23. $\frac{S_m}{S_n} = \frac{m^2}{n^2} \Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$ 1

$\Rightarrow \quad \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$ 1

Solving we get d = 2a 1

$\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$

$= \frac{2m-1}{2n-1}$ 1

24. Let the speed of stream be x km/hr.

$\therefore$ Speed of boat upstream = (15 – x) km/hr. $\frac{1}{2}$

Speed of boat downstream = (15 + x) km/hr. $\frac{1}{2}$

$\frac{30}{15-x} + \frac{30}{15+x} = 4\frac{1}{2} = \frac{9}{2}$ 1

$\Rightarrow \quad \frac{30(15+x+15-x)}{(15-x)(15+x)} = \frac{9}{2}$

$\Rightarrow \quad 200 = 225 - x^2$ 1

x = 5 (Rejecting – 5)

$\therefore$ Speed of stream = 5 km/hr 1

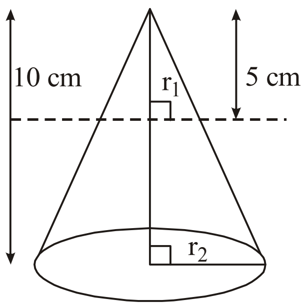
25. Area of traingle with vertices (a, a^2), (b, b^2) and $(0, 0)$ is

$$\frac{1}{2}|a(b^2)+b(-a^2)+0|$$ 2

$$=\frac{1}{2}ab(b-a)\neq 0 \quad \text{as} \quad a\neq b\neq 0$$ 2

$\therefore$ Given points are not collinear

26.



$$\frac{5}{10}=\frac{r_1}{r_2}$$

$\Rightarrow \quad r_2 = 2r_1$ 1

Ratio of volumes of two parts

$$=\frac{\text{Volume of smaller cone}}{\text{Volume of frustum}}$$

$$=\frac{\frac{1}{3}\pi\times r_1^2\times 5}{\frac{1}{3}\times\pi\times 5[r_1^2+r_2^2+r_1r_2]}=\frac{r_1^2}{r_1^2+4r_1^2+2r_1^2}$$ $1\frac{1}{2}+1$

$$=\frac{1}{7}$$ $\frac{1}{2}$

27. For Peter,

Total number of outcomes = 36

Favourable outcome is (5, 5)

$\therefore$ P(Peter getting the number 25) $=\frac{1}{36}$ $1\frac{1}{2}$

For Rina, Total number of outcomes = 6

Favourable outcome is 5.

$\therefore$ P (Rina getting the number 25) $=\frac{1}{6}$ $1\frac{1}{2}$

$\therefore$ Rina has the better chance 1

28. Area of minor segment

$$= \frac{22}{7} \times 10 \times 10 \times \frac{\cancel{60}^{1}}{\cancel{360}^{6}} - \frac{\sqrt{3}}{4} \times 10 \times 10$$

$$= 10 \times 10 \left[\frac{22}{7} \times \frac{1}{6} - \frac{\sqrt{3}}{4} \right]$$

$$= \frac{100}{84}(44 - 21\sqrt{3})\ \text{cm}^2 \quad \text{or} \quad \frac{25}{21}(44 - 21\sqrt{3})\ \text{cm}^2$$ $2\frac{1}{2}$

Area of major segment

$$= \left[\frac{22}{7} \times 10 \times 10 - \frac{100}{84}(44 - 21\sqrt{3}) \right] \text{cm}^2$$

$$= \frac{100}{84}(220 + 21\sqrt{3})\ \text{cm}^2 \quad \text{or} \quad \frac{25}{21}(220 + 21\sqrt{3})\ \text{cm}^2$$ $1\frac{1}{2}$

29.

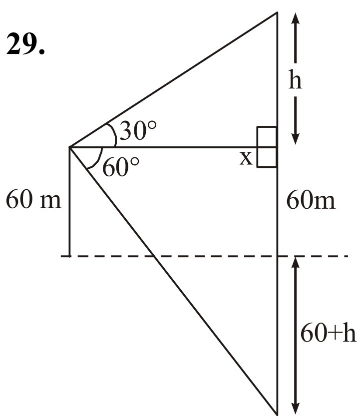


Figure 1

$$\frac{h}{x} = \tan 30° \Rightarrow x = h\sqrt{3}$$ 1

$$\frac{60 + 60 + h}{x} = \tan 60°$$

$$\Rightarrow \quad \frac{120 + h}{x} = \sqrt{3}$$ 1

$$\Rightarrow \quad 120 + h = h\sqrt{3} \times \sqrt{3}$$

$$\Rightarrow \quad h = 60$$ $\frac{1}{2}$

$\therefore$ height of cloud from surface of water = (60 + 60)m = 120 m $\frac{1}{2}$

30. Area of shaded region

= Area of square + Area of 2 major sectors. $1\frac{1}{2}$

$$= \left[28\times 28 + 2\times \frac{22}{7}\times 14\times 14\times \frac{270°}{360°}\right]\text{cm}^2$$ $1\frac{1}{2}$

$$= 28\times 28\left(1+\frac{33}{28}\right) = 1708\ \text{cm}^2$$ 1

31. Volume of water in cylindrical tank.

= Volume of water in park. 1

$\Rightarrow \quad \frac{22}{7}\times 1\times 1\times 5 = 25\times 20\times h$, where h is the height of standing water. $1\frac{1}{2}$

$\Rightarrow \quad h = \frac{11}{350}\ \text{m}\ \text{ or }\ \frac{22}{7}\ \text{cm}$ $\frac{1}{2}$

Conservation of water or any other relevant value. 1

QUESTION PAPER CODE 30/1/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

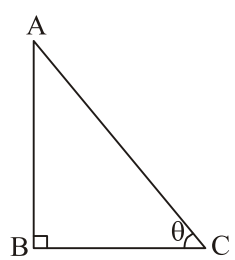
1. Favourable outcomes are –1, 0, 1 $\frac{1}{2}$

$\therefore$ Required Probability $= \frac{3}{7}$ $\frac{1}{2}$

2. $\sqrt{(4-1)^2 + (k-0)^2} = 5$ $\frac{1}{2}$

$\Rightarrow$ $k = \pm 4$ $\frac{1}{2}$

3.



$\tan\theta = \frac{AB}{BC} = \frac{\sqrt{3}}{1}$ $\frac{1}{2}$

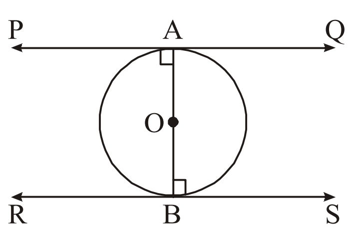
$\Rightarrow$ $\theta = 60°$ $\frac{1}{2}$

4. $\frac{2}{3}\pi r^3 = 3\pi r^2 \Rightarrow r = \frac{9}{2}$ units $\frac{1}{2}$

$\therefore$ d = 9 units $\frac{1}{2}$

SECTION B

5.



$\angle PAO = \angle OBS = 90°$ 1

But these are alternate interior angles

$\therefore$ PQ || RS 1

6. PA = PC + CA = PC + CQ

$\Rightarrow$ 12 = PC + 3 $\Rightarrow$ PC = 9 cm 1

PD = 9 cm

$\therefore$ PC + PD = 18 cm 1

7. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$ 1

$\Rightarrow \quad (\sqrt{2}\,x + 5)\,(x + \sqrt{2}) = 0$

$\Rightarrow \quad x = \frac{-5}{\sqrt{2}},\ -\sqrt{2}$

or $\quad \frac{-5\sqrt{2}}{2}, -\sqrt{2}$ 1

8. A.P. formed is 208, 216, 224, ..., 496 1

$a_n = 496$

$\Rightarrow \quad 208 + (n-1) \times 8 = 496$ $\frac{1}{2}$

$\Rightarrow \quad n = 37$ $\frac{1}{2}$

9. $x^2 + k(2x + k - 1) + 2 = 0$

$\Rightarrow \quad x^2 + 2kx + (k^2 - k + 2) = 0$ $\frac{1}{2}$

For equal roots, $b^2 - 4ac = 0$

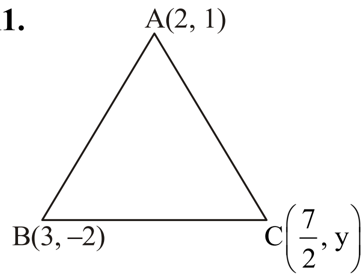
$\Rightarrow \quad 4k^2 - 4k^2 + 4k - 8 = 0$ 1

$\Rightarrow \quad k = 2$ $\frac{1}{2}$

10. Correct construction 2

SECTION C

11.



ar(ΔABC) = 5 sq.units

$\Rightarrow \quad \frac{1}{2}\left[2(-2-y)+3(y-1)+\frac{7}{2}(1+2)\right]=5$ $\qquad 1\frac{1}{2}$

$\Rightarrow \quad y+\frac{7}{2}=10$ $\qquad 1$

$\Rightarrow \quad y=\frac{13}{2}$ $\qquad \frac{1}{2}$

12. A(–2, 0), B(2, 0), C(0, 2)

AB = 4 units, BC = $2\sqrt{2}$ units, AC = $2\sqrt{2}$ units $\qquad 1$

P(–4, 0), Q(4, 0), R(0, 4)

PQ = 8 units, QR = $4\sqrt{2}$ units, PR = $4\sqrt{2}$ units $\qquad 1$

$\therefore \quad \frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}=\frac{1}{2}$ $\qquad 1$

$\therefore \quad \Delta ABC \sim \Delta PQR$

13. Total number of outcomes = 36

(i) Favourable outcomes are

(1, 1,) (1, 2) (1, 3) (1, 4) (1, 5) (2, 1) (2, 2) (2, 3)

(2, 4) (3, 1) (3, 2) (3, 3) (4, 1) (4, 2) (5, 1) i.e., 15

$\therefore \quad$ P(sum less than 7) = $\frac{15}{36}$ or $\frac{5}{12}$ $\qquad 1$

(ii) Favourable outcomes are

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3)

(2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (4, 1)

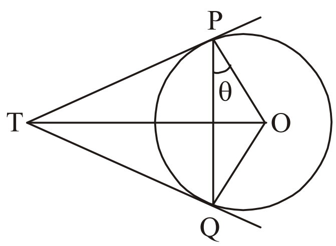
(4, 2) (4, 3) (5, 1) (5, 2) (5, 3) (6, 1) (6, 2) i.e., 25

P(product less than 16) $= \frac{25}{36}$ 1

(iii) Favourable outcomes are

$\therefore$ P(doublet of odd number) $= \frac{3}{36}$ or $\frac{1}{12}$ 1

14.



Let $\angle OPQ = \theta$

$\Rightarrow \quad \angle TPQ = 90° - \theta = \angle TQP$ 1

$\angle TPQ + \angle TQP + \angle PTQ = 180°$

$\Rightarrow \quad 90° - \theta + 90° - \theta + \angle PTQ = 180°$ $1\frac{1}{2}$

$\Rightarrow \quad \angle PTQ = 2\theta$

$= 2\angle OPQ$ $\frac{1}{2}$

15. $S_n = \left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ...$ upto n terms

$= \underbrace{(4 + 4 + ... + 4)}_{\text{n times}} - \frac{1}{n}(1 + 2 + 3 + ... + n)$ 1

$= 4n - \frac{1}{n} \times \frac{n(n+1)}{2}$ $\frac{1}{2}+1$

$= \frac{7n-1}{2}$ $\frac{1}{2}$

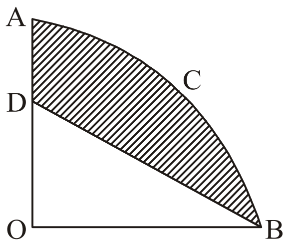
16. $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

For equal roots, $B^2 - 4AC = 0$ $\frac{1}{2}$

$\Rightarrow \quad 4m^2c^2 - 4(1 + m^2)(c^2 - a^2) = 0$ 1

$\Rightarrow \quad m^2c^2 - c^2 - m^2c^2 + a^2 + m^2a^2 = 0$ 1

$\Rightarrow \quad c^2 = a^2(1 + m^2)$ $\frac{1}{2}$

17. 

Area of shaded region = Area of quadrant OACB – Area of ΔODB 1

$= \left(\frac{22}{7} \times \frac{3.5 \times 3.5}{4} - \frac{1}{2} \times 3.5 \times 2\right) \text{cm}^2$ 1

$= \frac{49}{8}$ or 6.125 cm^2 1

18. $a_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$ ...(1) $\frac{1}{2}$

$a_n = \frac{1}{n} \Rightarrow a + (n-1)d = \frac{1}{m}$ ...(2) $\frac{1}{2}$

Solving (1) and (2) we get, $a = \frac{1}{mn}, d = \frac{1}{mn}$ 1

$a_{mn} = a + (mn - 1)d$

$= \frac{1}{mn} + (mn-1) \times \frac{1}{mn} = 1$ 1

19. Let the number of cones be n

Volume of solid sphere = Volume of n solid cones 1

$\Rightarrow \quad \frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5 = n \times \frac{1}{3} \times \pi \times 3.5 \times 3.5 \times 3$ 1

$\Rightarrow \quad n = \frac{4 \times 10.5 \times 10.5 \times 10.5}{3.5 \times 3.5 \times 3}$

$= 126.$ 1

20.

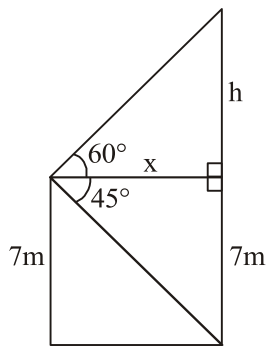


Figure $\frac{1}{2}$

$\frac{7}{x} = \tan 45°$

$\Rightarrow$ $x = 7$ m 1

$\frac{h}{x} = \tan 60°$

$h = x\sqrt{3}$

$= 7\sqrt{3}$ 1

$\therefore$ Height of tower $= (7\sqrt{3} + 7)$ m

$= 7(\sqrt{3} + 1)$ m $\frac{1}{2}$

SECTION D

21. Volume of water in cylindrical tank.

= Volume of water in park. 1

$\Rightarrow$ $\frac{22}{7} \times 1 \times 1 \times 5 = 25 \times 20 \times h$, where h is the height of standing water. $1\frac{1}{2}$

$\Rightarrow$ $h = \frac{11}{350}$ m or $\frac{22}{7}$ cm $\frac{1}{2}$

Conservation of water or any other relevant value. 1

22. Area of shaded region

= Area of square + Area of 2 major sectors. $1\frac{1}{2}$

$= \left[28 \times 28 + 2 \times \frac{22}{7} \times 14 \times 14 \times \frac{270°}{360°}\right] \text{cm}^2$ $1\frac{1}{2}$

$= 28 \times 28\left(1 + \frac{33}{28}\right) = 1708 \text{ cm}^2$ 1

23. For Peter,

Total number of outcomes = 36

Favourable outcome is (5, 5)

$\therefore$ P(Peter getting the number 25) = $\frac{1}{36}$ $1\frac{1}{2}$

For Rina, Total number of outcomes = 6

Favourable outcome is 5.

$\therefore$ P (Rina getting the number 25) = $\frac{1}{6}$ $1\frac{1}{2}$

$\therefore$ Rina has the better chance 1

24. Area of minor segment

$$= \frac{22}{7} \times 10 \times 10 \times \frac{\cancel{60}^{1}}{\cancel{360}^{6}} - \frac{\sqrt{3}}{4} \times 10 \times 10$$

$$= 10 \times 10 \left[\frac{22}{7} \times \frac{1}{6} - \frac{\sqrt{3}}{4} \right]$$

$$= \frac{100}{84}(44 - 21\sqrt{3})\ \text{cm}^2 \quad \text{or} \quad \frac{25}{21}(44 - 21\sqrt{3})\ \text{cm}^2$$ $2\frac{1}{2}$

Area of major segment

$$= \left[\frac{22}{7} \times 10 \times 10 - \frac{100}{84}(44 - 21\sqrt{3}) \right] \text{cm}^2$$

$$= \frac{100}{84}(220 + 21\sqrt{3})\ \text{cm}^2 \quad \text{or} \quad \frac{25}{21}(220 + 21\sqrt{3})\ \text{cm}^2$$ $1\frac{1}{2}$

25. Correct figure, given, to prove and construction $\frac{1}{2} \times 4 = 2$

Correct proof 2

26. Let the speed of stream be x km/hr.

$\therefore$ Speed of boat upstream = (15 – x) km/hr. $\frac{1}{2}$

Speed of boat downstream = (15 + x) km/hr. $\frac{1}{2}$

$$\frac{30}{15-x} + \frac{30}{15+x} = 4\frac{1}{2} = \frac{9}{2}$$ 1

$\Rightarrow$ $\frac{30(15+x+15-x)}{(15-x)(15+x)} = \frac{9}{2}$

$\Rightarrow$ $200 = 225 - x^2$ 1

x = 5 (Rejecting – 5)

$\therefore$ Speed of stream = 5 km/hr 1

27. Area of traingle with vertices (a, a^2), (b, b^2) and (0, 0) is

$$\frac{1}{2}|a(b^2) + b(-a^2) + 0|$$ 2

$$= \frac{1}{2}ab(b-a) \neq 0 \quad \text{as} \quad a \neq b \neq 0$$ 2

$\therefore$ Given points are not collinear

28. Correct construction of given right triangle 2

Corect construction of similar triangle 2

29. $S_m = S_n \Rightarrow \frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$ 1

$\Rightarrow$ $2a(m-n) = -d(m+n-1)(m-n)$

$\Rightarrow$ $2a + d(m+n-1) = 0$ 2

$$S_{m+n} = \frac{m+n}{2}[2a + d(m+n-1)] = 0$$ 1

30.

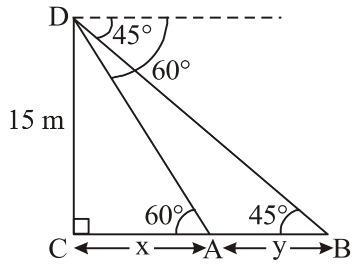


Figure 1

$$\frac{15}{x} = \tan 60°$$

$\Rightarrow \quad x = \frac{15}{\sqrt{3}} = 5\sqrt{3}$ m 1

$$\frac{15}{x+y} = \tan 45°$$

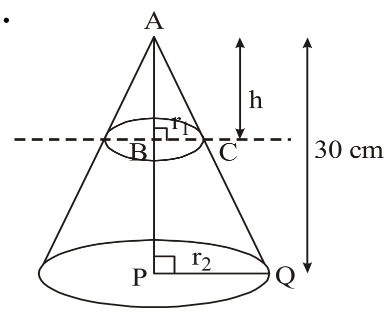
$\Rightarrow \quad 15 = 5\sqrt{3} + y$ 1

$\Rightarrow \quad y = 15 - 5\sqrt{3}$

$= 5(3 - \sqrt{3})$

$\therefore$ Distance between two points $= 5(3 - \sqrt{3})$ m 1

31.



$\Delta ABC \sim \Delta APQ$

$\Rightarrow \quad \frac{h}{30} = \frac{r_1}{r_2} \quad$ ...(i) 1

Volume of smaller cone

$= \frac{1}{27} \times$ Volume of larger cone

$\Rightarrow \quad \frac{1}{3}\pi r_1^2 \times h = \frac{1}{27} \times \frac{1}{3}\pi r_2^2 \times 30$ 1

$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^2 \times \frac{h}{30} = \frac{1}{27}$

$\Rightarrow \quad \left(\frac{h}{30}\right)^3 = \frac{1}{27} \quad$ (using (i)) 1

$\Rightarrow \quad h = 10$ cm $\frac{1}{2}$

$\therefore \quad$ Required height $= (30 - 10)$ cm $= 20$ cm $\frac{1}{2}$

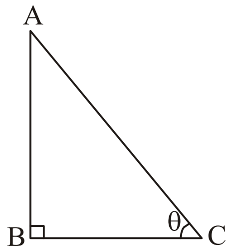
QUESTION PAPER CODE 30/1/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\sqrt{(4-1)^2 + (k-0)^2} = 5$ $\frac{1}{2}$

$\Rightarrow$ $k = \pm 4$ $\frac{1}{2}$

2. 

$\tan\theta = \frac{AB}{BC} = \frac{\sqrt{3}}{1}$ $\frac{1}{2}$

$\Rightarrow$ $\theta = 60°$ $\frac{1}{2}$

3. $\frac{2}{3}\pi r^3 = 3\pi r^2 \Rightarrow r = \frac{9}{2}$ units $\frac{1}{2}$

$\therefore$ $d = 9$ units $\frac{1}{2}$

4. Favourable outcomes are –1, 0, 1 $\frac{1}{2}$

$\therefore$ Required Probability $= \frac{3}{7}$ $\frac{1}{2}$

SECTION B

5. PA = PC + CA = PC + CQ

$\Rightarrow$ 12 = PC + 3 $\Rightarrow$ PC = 9 cm 1

PD = 9 cm

$\therefore$ PC + PD = 18 cm 1

6. Correct construction 2

7. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$ 1

$\Rightarrow \quad (\sqrt{2}\,x + 5)(x + \sqrt{2}) = 0$

$\Rightarrow \quad x = \dfrac{-5}{\sqrt{2}}, -\sqrt{2}$

or $\quad \dfrac{-5\sqrt{2}}{2}, -\sqrt{2}$ 1

8. A.P. formed is 208, 216, 224, ..., 496 1

$a_n = 496$

$\Rightarrow \quad 208 + (n - 1) \times 8 = 496$ $\frac{1}{2}$

$\Rightarrow \quad n = 37$ $\frac{1}{2}$

9. $x^2 + k(2x + k - 1) + 2 = 0$

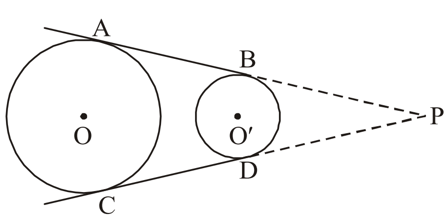
$\Rightarrow \quad x^2 + 2kx + (k^2 - k + 2) = 0$ $\frac{1}{2}$

For equal roots, $b^2 - 4ac = 0$

$\Rightarrow \quad 4k^2 - 4k^2 + 4k - 8 = 0$ 1

$\Rightarrow \quad k = 2$ $\frac{1}{2}$

10.



Construction: Extend AB and CD to meet at P $\frac{1}{2}$

$PA = PC$ $\frac{1}{2}$

$PB = PD$

$\Rightarrow \quad PA - PB = PC - PD$ $\frac{1}{2}$

$\Rightarrow \quad AB = CD$ $\frac{1}{2}$

SECTION C

11. Total number of outcomes = 36

(i) Favourable outcomes are

(1, 1,) (1, 2) (1, 3) (1, 4) (1, 5) (2, 1) (2, 2) (2, 3)

(2, 4) (3, 1) (3, 2) (3, 3) (4, 1) (4, 2) (5, 1) i.e., 15

$\therefore$ P(sum less than 7) = $\frac{15}{36}$ or $\frac{5}{12}$ 1

(ii) Favourable outcomes are

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3)

(2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (4, 1)

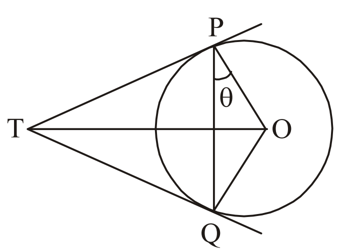
(4, 2) (4, 3) (5, 1) (5, 2) (5, 3) (6, 1) (6, 2) i.e., 25

P(product less than 16) = $\frac{25}{36}$ 1

(iii) Favourable outcomes are

$\therefore$ P(doublet of odd number) = $\frac{3}{36}$ or $\frac{1}{12}$ 1

12.



Let $\angle OPQ = \theta$

$\Rightarrow \quad \angle TPQ = 90° - \theta = \angle TQP$ 1

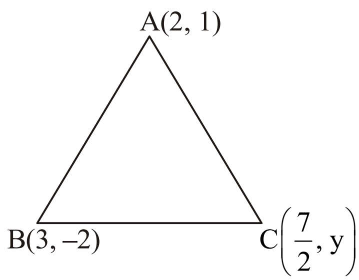
$\angle TPQ + \angle TQP + \angle PTQ = 180°$

$\Rightarrow \quad 90° - \theta + 90° - \theta + \angle PTQ = 180°$ $1\frac{1}{2}$

$\Rightarrow \quad \angle PTQ = 2\theta$

$= 2\angle OPQ$ $\frac{1}{2}$

13.



ar(ΔABC) = 5 sq.units

$\Rightarrow \quad \frac{1}{2}\left[2(-2-y)+3(y-1)+\frac{7}{2}(1+2)\right] = 5$ $1\frac{1}{2}$

$\Rightarrow \quad y + \frac{7}{2} = 10$ 1

$\Rightarrow \quad y = \frac{13}{2}$ $\frac{1}{2}$

14. $S_n = \left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ...$ upto n terms

$= \underbrace{(4 + 4 + ... + 4)}_{\text{n times}} - \frac{1}{n}(1 + 2 + 3 + ... + n)$ 1

$= 4n - \frac{1}{n} \times \frac{n(n+1)}{2}$ $\frac{1}{2}+1$

$= \frac{7n-1}{2}$ $\frac{1}{2}$

15. A(–2, 0), B(2, 0), C(0, 2)

AB = 4 units, BC = $2\sqrt{2}$ units, AC = $2\sqrt{2}$ units 1

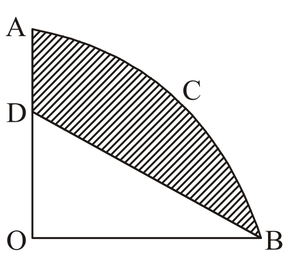
P(–4, 0), Q(4, 0), R(0, 4)

PQ = 8 units, QR = $4\sqrt{2}$ units, PR = $4\sqrt{2}$ units 1

$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{1}{2}$ 1

$\therefore \quad \Delta ABC \sim \Delta PQR$

16.



Area of shaded region = Area of quadrant OACB – Area of ΔODB 1

$= \left(\frac{22}{7} \times \frac{3.5 \times 3.5}{4} - \frac{1}{2} \times 3.5 \times 2\right) \text{cm}^2$ 1

$= \frac{49}{8}$ or 6.125 cm^2 1

17. $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

For equal roots, $B^2 - 4AC = 0$ $\frac{1}{2}$

$\Rightarrow \quad 4m^2c^2 - 4(1 + m^2)(c^2 - a^2) = 0$ 1

$\Rightarrow \quad m^2c^2 - c^2 - m^2c^2 + a^2 + m^2a^2 = 0$ 1

$\Rightarrow \quad c^2 = a^2(1 + m^2)$ $\frac{1}{2}$

18. $a_p = q \Rightarrow a + (p-1)d = q \quad ...(1)$ $\frac{1}{2}$

$a_q = p \Rightarrow a + (q-1)d = p \quad ...(2)$ $\frac{1}{2}$

Solving (1) and (2) we get, $a = p + q - 1,\ d = -1$ 1

$a_n = a + (n-1)d$

$= (p + q - 1) + (n-1)(-1)$

$= p + q - n$ 1

19. Let the number of cones be n

Volume of sphere = Volume of n cones 1

$\Rightarrow \quad \frac{4}{3}\pi \times 8 \times 8 \times 8 = n \times \frac{1}{3} \times \pi \times 4 \times 4 \times 8$ 1

$\Rightarrow \quad n = \frac{4 \times 8 \times 8 \times 8}{4 \times 4 \times 8}$

$= 16$ 1

20.

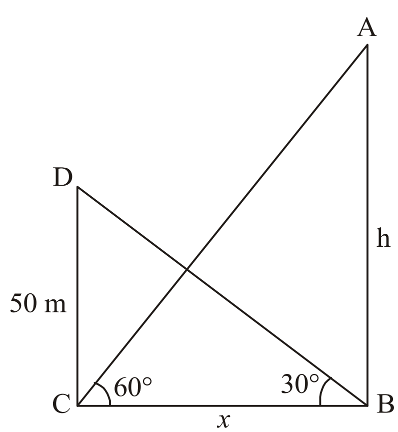


Figure $\frac{1}{2}$

$\frac{50}{x} = \tan 30°$

$\Rightarrow \quad x = 50\sqrt{3}$ 1

$\frac{h}{x} = \tan 60°$

$\Rightarrow \quad h = 50\sqrt{3} \times \sqrt{3} = 150$ 1

$\therefore$ Height of hill = 150 m $\frac{1}{2}$

SECTION D

21. Area of shaded region

= Area of square + Area of 2 major sectors. $1\frac{1}{2}$

$= \left[28 \times 28 + 2 \times \frac{22}{7} \times 14 \times 14 \times \frac{270°}{360°}\right] \text{cm}^2$ $1\frac{1}{2}$

$= 28 \times 28\left(1 + \frac{33}{28}\right) = 1708 \text{ cm}^2$ 1

22. Volume of water in cylindrical tank.

= Volume of water in park. 1

$\Rightarrow \frac{22}{7} \times 1 \times 1 \times 5 = 25 \times 20 \times h$, where h is the height of standing water. $1\frac{1}{2}$

$\Rightarrow h = \frac{11}{350}$ m or $\frac{22}{7}$ cm $\frac{1}{2}$

Conservation of water or any other relevant value. 1

23. Area of minor segment

$= \frac{22}{7} \times 10 \times 10 \times \frac{\cancel{60}^{1}}{\cancel{360}^{6}} - \frac{\sqrt{3}}{4} \times 10 \times 10$

$= 10 \times 10\left[\frac{22}{7} \times \frac{1}{6} - \frac{\sqrt{3}}{4}\right]$

$= \frac{100}{84}(44 - 21\sqrt{3}) \text{ cm}^2$ or $\frac{25}{21}(44 - 21\sqrt{3}) \text{ cm}^2$ $2\frac{1}{2}$

Area of major segment

$= \left[\frac{22}{7} \times 10 \times 10 - \frac{100}{84}(44 - 21\sqrt{3})\right] \text{cm}^2$

$= \frac{100}{84}(220 + 21\sqrt{3}) \text{ cm}^2$ or $\frac{25}{21}(220 + 21\sqrt{3}) \text{ cm}^2$ $1\frac{1}{2}$

24. For Peter,

Total number of outcomes = 36

Favourable outcome is (5, 5)

$\therefore$ P(Peter getting the number 25) = $\frac{1}{36}$ $1\frac{1}{2}$

For Rina, Total number of outcomes = 6

Favourable outcome is 5.

$\therefore$ P (Rina getting the number 25) = $\frac{1}{6}$ $1\frac{1}{2}$

$\therefore$ Rina has the better chance 1

25. Correct figure, given, to prove and construction $\frac{1}{2}\times4=2$

Correct proof 2

26. Let the speed of stream be x km/hr.

$\therefore$ Speed of boat upstream = (15 – x) km/hr. $\frac{1}{2}$

Speed of boat downstream = (15 + x) km/hr. $\frac{1}{2}$

$$\frac{30}{15-x}+\frac{30}{15+x} = 4\frac{1}{2} = \frac{9}{2}$$ 1

$$\Rightarrow \frac{30(15+x+15-x)}{(15-x)(15+x)} = \frac{9}{2}$$

$\Rightarrow$ $200 = 225 - x^2$ 1

x = 5 (Rejecting – 5)

$\therefore$ Speed of stream = 5 km/hr 1

27. Area of traingle with vertices (a, a^2), (b, b^2) and (0, 0) is

$$\frac{1}{2}|a(b^2)+b(-a^2)+0|$$ 2

$= \frac{1}{2}ab(b-a) \neq 0$ as $a \neq b \neq 0$ 2

$\therefore$ Given points are not collinear

28. Correct construction of ΔABC 2

Correct construction of similar triangle 2

29. $a_p = \frac{1}{q} \Rightarrow a + (p-1)d = \frac{1}{q}$ ...(1) 1

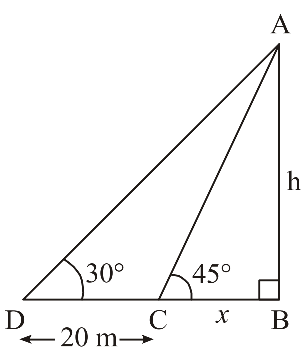
$a_q = \frac{1}{p} \Rightarrow a + (q-1)d = \frac{1}{p}$ ...(2) 1

Solving (1) and (2) we get, $a = \frac{1}{pq}, d = \frac{1}{pq}$ 1

$$S_{pq} = \frac{pq}{2}\left[2 \times \frac{1}{pq} + (pq-1) \times \frac{1}{pq}\right]$$

$= \frac{(pq+1)}{2}$ 1

30. Figure 1



$\frac{h}{x} = \tan 45°$

$\Rightarrow$ $h = x$ 1

$\frac{h}{x+20} = \tan 30°$

$\Rightarrow$ $h\sqrt{3} = x + 20$

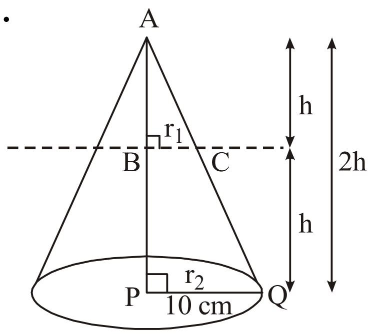
$\Rightarrow$ $h\sqrt{3} = h + 20$ 1

$\Rightarrow$ $h = \frac{20}{\sqrt{3}-1}$

or $10(\sqrt{3}+1)$ 1

$\therefore$ Height of tower $= 10(\sqrt{3}+1)$ m

31.



$\Delta ABC \sim \Delta APQ$

$\Rightarrow \quad \frac{h}{2h} = \frac{r_1}{10}$

$\Rightarrow \quad r_1 = 5$ cm 1

Volume of smaller cone

$= \frac{1}{3}\pi(5)^2 \times h$ 1

Volume of frustum

$= \frac{1}{2}\pi \times h\,(5^2 + 10^2 + 5 \times 10)$

$= \frac{1}{3}\pi \times h \times 175$ $1\frac{1}{2}$

Required ratio $= \dfrac{\frac{1}{3} \times \pi \times 25 \times h}{\frac{1}{3} \times \pi \times h \times 175} = \frac{1}{7}$ $\frac{1}{2}$